M463 Homework 12

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Suppose you have $\$100,000$ to invest in stocks. If you invest $\$1000$ in any particular stock your profit will be $\$200, \$100$ or $\$100$ (a loss), with probability 0.25 each. There are 100 different stocks you can choose from, and they all behave independently of each other. Consider the two cases: (1) Invest $\$100,000$ in one stock. (2) Invest $\$1000$ in each of 100 stocks.

a) For case (1) find the probability that your profit will be $8000 or more.

Solution: Let X = profit in one particular stock. The following table summarizes the data for X.

x	$P(X=x)$	$xP(X=x)$	$x^2P(X=x)$
200	1/4	50	10,000
100	1/4	25	2,500
0	1/4	0	0
−100	1/4	−25	2,500

Hence, $E(X) = 50$ and $Var(X) = 15,000 - 2,500 = 12,500 \Rightarrow S.D.(X) = 111.8033989$.
Note that If you invest $\$100,000$ in one particular stock, this is equivalent to buying 100 shares of that stock. Your profit is then giving by $Y = 100X$ = profit on 100 shares of one particular stock.
Y is completely given by X

x	y	$P(Y=y)$
200	20,000	1/4
100	10,000	1/4
0	0	1/4
−100	−10,000	1/4

So, $P(Y \geq 8,000) = P(Y = 10,000 \text{ or } Y = 20,000) = P(Y = 10,000) + P(20,000) = \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$

b) Do the same for case (2).

Solution: Let $S_{100} = X_1 + X_2 + \cdots + X_{100}$ be the profit in 100 different, independent stocks.
We can approximate the probability $P(S_{100} \geq 8000)$ using the normal distribution. By the Central Limit Theorem, S_{100} is approximately normal with mean $E(S_{100}) = 100 \cdot E(X_i) = 100 \cdot 50 = 5,000$ and standard deviation $S.D(S_{100}) = \sqrt{100}SD(X_i) = 10 \cdot 111.8033989 = 1,118.033989$. Hence:

$$\begin{aligned} P(S_{100} \geq 8,000) = 1 - P(S_{100} < 8,000) &= 1 - P\left(\frac{S_{100} - nE(X_i)}{\sqrt{n}SD(X_i)} < \frac{8,000 - 5,000}{1,118.033989}\right) \\ &\approx 1 - P(Z \leq 2.683281573) \\ &= 1 - \Phi(2.683281573) = \boxed{0.003645179} \end{aligned}$$